Scope and Sequence
Grade K

## Module Objectives

| 1 | 1. Learn the routines and expectations for ST Math Summer Immersion <br> 2. Addition within 10 and comparing numbers. | K.OA.A. 1 <br> K.CC.C. 6 | Students: <br> - will develop an understanding of the Problem-Solving Process and apply this thinking as a strategy for solving puzzles in ST Math. <br> - will be engaged in conversations and practice that will teach them how to utilize the strategy. <br> - will develop an understanding of the design process and understand the purpose and expectations of the stations. <br> - develop their understanding of solving addition problems within 10 . Students will write equations to represent the problems. <br> - will solve addition word problems. |
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| 2 | 1. Addition within 10. <br> 2. Writing equations <br> 3. Solve word problems <br> 4. Compare numbers <br> 5. Math vocabulary <br> 6. One or two more or less strategies for adding | K.OA.A. 2 <br> K.OA.A. 5 <br> K.CC.C. 7 | Students: <br> - develop their understanding of solving addition problems within 10. <br> - will write equations to represent the problems. <br> - will solve addition word problems. |
| 3 | 1. Subtraction within 10 <br> 2. Writing equations <br> 3. Solve word problems <br> 4. Math vocabulary | K.OA.A. 2 <br> K.OA.A. 5 | Students: <br> - develop their understanding of solving subtraction problems within 10. <br> - will write equations to represent the problems. <br> - will solve subtraction word problems. |
| 4 | 1. Making 10 . <br> 2. Writing equations <br> 3. Solve word problems | $\text { K.OA.A. } 3$ $\text { K.OA.A. } 4$ | Students: <br> - will use different models to compose 10 when one addend is given. <br> - will solve problems involving making combinations of 10. <br> - will decompose numbers less than or equal to 10 into different pairs of addends. <br> - solve word problems involving making 10. |
| 5 | 1. Decompose numbers to 10 <br> 2. Writing equations <br> 3. Solve word problems | K.OA.A. 3 K.OA.A. 4 | Students: <br> - will use different models to compose 10 when one addend is given. <br> - will solve problems involving making combinations of 10. <br> - will decompose numbers less than or equal to 10 into different pairs of addends. <br> - solve word problems involving making 10. |

## Ongoing Math Focus

## Student Discourse

## Students discuss:

- discuss the strategies they use to solve the puzzles and problems.
- discuss and the teacher charts the math concepts and vocabulary.
- discuss what the drawings or manipulatives represent in the puzzle or problem.
- discuss what the numbers in their equation represent in the puzzle or problem.
- discuss how the jumps on a number line represent addition.


## Student Problem-Solving Strategies

## Students:

- compare the number of objects in two groups.
- discuss what is known and what is unknown in the puzzle and problem.
- write all of the combinations that will make numbers up to and including 10.
- answer questions that focus on one or two more or less strategies for adding and subtracting.
- answer questions about how a different number will change the solution to a problem.


## Representations

## Students:

- represent the puzzles on a game mat using manipulatives, drawings, or numbers to help them solve problems.
- draw pictures or write numerals on blank paper to represent the puzzle and show the solution.
- represent problems with numbers and symbols.
- write equations to represent the problem and solution.
- learn to use one ten frame to organize counters and examine the relationship of numbers using benchmark numbers 5 and 10.
- use two empty ten frames to help them solve the puzzles and problems.
- represent equations, puzzles, and problems as jumps on a number line.


## Standards

## Counting and Cardinality

K.CC.C. 7 Compare two numbers between 1 and 10 presented as written numerals.

## Operations and Algebraic Thinking

K.OA.A. 1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
K.OA.A. 2 Solve addition and subtraction word problems and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
K.OA.A. 3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5=2+3$ and $5=4+1$ ).
K.OA.A. 4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
K.OA.A. 5 Fluently add and subtract within 5 .
K.CC.C. 6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

Scope and Sequence
Grade 1

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 1 | 1.Learn the routines and expectations for ST Math Summer Immersion <br> 2.Addition and subtraction of whole numbers <br> 3.Write equations to represent a problem <br> 4.Determine if equations are true or false | 1.OA.A. 1 <br> 1.OA.C. 5 <br> 1.OA.C. 6 <br> 1.OA.D. 7 | Students: <br> - will develop an understanding of the Problem Solving Process and apply this thinking as a strategy for solving puzzles in ST Math. <br> - will be engaged in conversations and practice that will teach them how to utilize the strategy. <br> - will develop an understanding of the design process and understand the purpose and expectations of the stations. <br> - will begin solving problems involving addition and subtraction and writing equations to represent the problems. |
| 2 | 1.Solve word problems involving addition and subtraction. <br> 2.Meaning of the equal sign | 1.OA.A. 1 <br> 1.OA.A. 2 <br> 1.OA.B. 4 <br> 1.OA.D. 7 <br> 1.OA.D. 8 | Students: <br> - work with puzzles involving addition and subtraction with the result, change, or start unknown. <br> - determine what is known and unknown in a puzzle or problem. <br> - write equations to represent the problems. <br> - discuss the meaning of the equal sign and whether equations are true or false. <br> - solve addition and subtraction word problems. |
| 3 | 1.Writing equations to solve problems with addition and subtraction. | 1.OA.A. 1 <br> 1.OA.A. 2 <br> 1.OA.B. 4 <br> 1.OA.D. 7 <br> 1.OA.D. 8 | Students: <br> - work with puzzles involving addition and subtraction with the result, change, or start unknown. <br> - determine the unknown. <br> - write equations to represent the problems. <br> - discuss the meaning of the equal sign and whether equations are true or false. <br> - solve addition and subtraction word problems. <br> - add ten to a solution of a problem and write an equation to show the new total. |
| 4 | 1.Determining the unknown whole number in an addition or subtraction equation. | $\begin{aligned} & \text { 1.OA.A. } 1 \\ & \text { 1.OA.B. } 4 \\ & \text { 1.OA.D. } 7 \end{aligned}$ | Students: <br> - work with puzzles involving addition and subtraction with the result, change, or start unknown. <br> - determine the unknown. <br> - write equations to represent the problems. <br> - discuss the meaning of the equal sign and whether equations are true or false. <br> - solve addition and subtraction word problems. |


| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 5 | 1.Using the relationship between addition and subtraction to solve problems. | 1.OA.A. 1 <br> 1.OA.B. 3 <br> 1.OA.B. 4 <br> 1.OA.C. 5 | Students: <br> - work with puzzles involving missing addends. <br> - write equations to represent the problems. <br> - discuss the meaning of the equal sign and whether equations are true or false. <br> - discuss the relationship between addition and subtraction. <br> - explore the commutative property of addition. |


| Ongoing Math Focus: |  |  |
| :---: | :---: | :---: |
| Student Discourse | Student Problem-Solving Strategies | Representations |
| Students discuss: <br> - how different combinations of creatures can equal the same number of shoes. <br> - what the numbers in the equations represent in the puzzles. <br> - whether equations are true or false. <br> - what is known and unknown in the puzzle and problem. <br> - the action in the puzzle when it is solved. <br> - multiple solutions for one puzzle or problem. <br> - the relationship of addition and subtraction. <br> - using benchmarks of five and ten to help add and subtract. <br> - commutative property of addition. | Students: <br> - determine what is known and unknown in a problem. <br> - find all the possible solutions to a puzzle. <br> - chart the combinations that make 5 and the combinations that make 10. <br> - write an equation for the combinations to make the number. <br> - use benchmarks of five and ten to help add and subtract. <br> - discuss that for this situation in the puzzle, order matters. <br> - use the relationship of addition and subtraction to solve a puzzle or problem. | Students: <br> - will use the Creature Cards to help represent and solve the puzzles. <br> - will write equations to show that two different solutions are equal. <br> - represent the puzzles on a game mat using manipulatives, drawings, or numbers to help them solve problems. <br> - show what the puzzle would look like with visuals instead of numbers. <br> - write equations to represent the puzzles including a symbol for the unknown in the problem. <br> - write and discuss a story problem for one of the puzzles. |

## Standards

## Operations and Algebraic Thinking

## Represent and solve problems involving addition and subtraction.

1.OA.A. 1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA.A. 2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 , e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.
1.OA.B.3 Apply properties of operations as strategies to add and subtract. 3 Examples: If $8+3=11$ is known, then $3+8=11$ is also known.
(Commutative property of addition.) To add $2+6+4$, the second two numbers can be added to make a ten, so $2+6+4=2+10=12$. (Associative property of addition.)
1.OA.B.4 Understand subtraction as an unknown-addend problem. (e.g., subtract $10-8$ by finding the number that makes 10 when added to 8. .)

## Add and subtract within 20.

1.OA.C. 5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2 ).
1.OA.C. 6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8+6=8+2+4=10+4=14$ ); decomposing a number leading to a ten (e.g., $13-4=13-3-1=10-1=9$ ); using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $12-8=4$ ); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+1=12+1=13)$.

## Work with addition and subtraction equations.

1.OA.D.7 Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6=6,7=8-1,5+2=2+5,4+1=5+2$.
1.OA.D. 8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8+?=11,5=-3,6+6=\beta$.

Scope and Sequence
Grade 2

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 1 | 1. Learn the routines and expectations for ST Math Summer Immersion <br> 2. Addition and subtraction of whole numbers | $\begin{aligned} & \text { 2.OA.A. } 1 \\ & \text { 2.OA.B. } 2 \end{aligned}$ | Students: <br> - will develop an understanding of the Problem-Solving Process and apply this thinking as a strategy for solving puzzles in ST Math. <br> - will be engaged in conversations and practice that will teach them how to utilize the strategy. <br> - will develop an understanding of the design process and understand the purpose and expectations of the stations. <br> - will begin solving problems involving addition and subtraction within 100. <br> - will write equations to represent the problems including equations with a symbol for the unknown. |
| 2 | 1. Solving problems involving addition and subtraction | $\begin{aligned} & \text { 2.OA.A. } 1 \\ & \text { 2.OA.B. } 2 \end{aligned}$ | Students: <br> - develop their understanding of addition and subtraction situations within 100 to solve problems. <br> - use strategies involving situations of adding to, taking from, putting together, taking apart, and comparing unknowns in different positions. <br> - will find missing addends. <br> - decompose numbers to make adding easier <br> - practice adding one-digit and two-digit whole numbers. <br> - will represent situations with equations. |
| 3 | 1. Solving one and two-step problems involving addition and subtraction within 100 <br> 2. Solving problems involving equal groups <br> 3. Even and odd numbers | $\begin{aligned} & \text { 2.OA.A. } 1 \\ & \text { 2.OA.B. } 2 \end{aligned}$ | Students: <br> - work with puzzles to develop their understanding of addition and subtraction situations within 100 to solve one-step and two-step problems. <br> - use strategies involving situations of adding to, taking from, putting together, taking apart, and comparing unknowns in different positions. <br> - will represent situations with equations. <br> - work with puzzles to develop their understanding of equal groups. <br> - will solve problems situations involving array and equal group models. <br> - will represent solutions with equations to express the total as a sum of equal addends. <br> - will identify multiplication expressions as equivalent to repeated addition. This will lay a foundation for the understanding of multiplication. <br> - will also discuss the characteristics of even and odd numbers. |

Scope and Sequence
Grade 2

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 4 | 1. Solving problems involving equal groups <br> 2. Solving problems using arrays and repeated addition <br> 3. Even and odd numbers | $\begin{aligned} & \text { 2.OA.C. } 3 \\ & \text { 2.OA.C. } 4 \end{aligned}$ | Students: <br> - work with puzzles to develop their understanding of equal groups. <br> - will solve problems situations involving array and equal group models. <br> - will represent solutions with equations to express the total as a sum of equal addends. <br> - will identify multiplication expressions as equivalent to repeated addition. <br> - will lay a foundation for the understanding of multiplication. <br> - will also discuss the characteristics of even and odd numbers. |
| 5 | 1. Representing numbers with repeated addition | $\begin{aligned} & \text { 2.OA.C. } 3 \\ & \text { 2.OA.C. } \end{aligned}$ | Students: <br> - work with arrays to represent repeated addition. <br> - will find multiple ways to represent a number. |

## Ongoing Math Focus

| Student Discourse | Student Problem-Solving Strategies | Representations |
| :---: | :---: | :---: |
| Students discuss: <br> - and chart the vocabulary and the math that is happening in the puzzles. <br> - the relationship of numbers and how a bar model and the number line show these relationships. <br> - how the numbers can be decomposed as jumps on a number line and how that helps with addition and subtraction of multi-digit numbers. <br> - different strategies students use to solve the problem. <br> - and show the action in the puzzles. <br> - the mathematics students are using to solve the puzzles and problems. | Students: <br> - share their solutions to problems and critique other students' solution strategies. <br> - determine how the numbers can be decomposed as jumps on a number line to help with the addition and subtraction of multi-digit numbers. <br> - use game mats to help solve the puzzles. <br> - use the Creature Cards to find different solutions to the puzzles. <br> - try a solution that has the correct number of squares but not with the correct number of rows or columns. Discuss why this does not work. | Students: <br> - write equations to represent the solutions. <br> - write equations with a symbol or letter to represent the missing addend. <br> - show the problem as jumps on a number line. <br> - continue to represent the puzzles in multiple ways, equations, bar models, and as jumps on a number line. <br> - represent the puzzles visually on their game mats and with an equation. <br> - compare their solution on the game mat to arrays. <br> - create an array with centimeter cubes to model a puzzle. <br> - write equations using repeated addition to represent the puzzles. |

## Student Discourse

Student Problem-Solving Strategies
Representations

- what the solution would look like on a number line.
- what they know about arrays and how the array models the puzzle or problem.
- and compare the mathematics in two different games.
- patterns they notice.


## Grade 2

## Standards

## Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.
2.OA.A. 1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.
2.OA.B. 2 Fluently add and subtract within 20 using mental strategies. ${ }^{2}$ By the end of Grade 2 , know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.
2.OA.C. 3 Determine whether a group of objects (up to 20 ) has an odd or even number of members, e.g., by pairing objects or counting them by 2 ; write an equation to express an even number as a sum of two equal addends.
2.OA.C. 4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Scope and Sequence
Grade 3

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 1 | 1. Learn the routines and expectations for ST Math Summer Immersion <br> 2. Begin working with problems involving fractions. <br> 3. Compare fractions and count by unit fractions. | 3.NF.A. 1 | Students: <br> - will develop an understanding of the Problem-Solving Process and apply this thinking as a strategy for solving puzzles in ST Math. <br> - will be engaged in conversations and practice that will teach them how to utilize the strategy. <br> - will develop an understanding of the design process and understand the purpose and expectations of the stations. <br> - will begin solving problems involving fractions and understanding fractions as numbers, including counting by unit fractions. |
| 2 | 1. Solving problems involving partitioning a whole into equal sections (fractions). | 3.NF.A. 1 | Students: <br> - use their understanding of fair sharing whole numbers into equal groups to partition one whole into equal fractional sections. <br> - partition a whole into halves, thirds, fourths, and eighths. <br> - understand that a fraction $1 / b$ is the quantity formed by 1 part of a whole partitioned into $b$ equal parts. <br> - work with puzzles involving selecting $a$ fraction of size $1 / b$ to create a fraction $a / b$. |
| 3 | 1. Defining and partitioning one (whole) and combining unit fractions. | 3.NF.A. 1 <br> 3.NF.A.2.a <br> 3.NF.A.2.b | Students: <br> - work with puzzles involving partitioning a rectangle into equal areas. <br> - understand that a fraction $1 / b$ is the quantity formed by 1 part of a whole partitioned into b equal parts and combine the unit fractions to create $\mathrm{a} / \mathrm{b}$ fractions. <br> - work with puzzles involving representing fractions as a bar model, moving the bar model to a number line, and understanding a fraction as a number on a number line. <br> - represent a fraction $1 / \mathrm{b}$ on a number line by defining the interval from 0 to 1 and partitioning it into b equal parts. <br> - recognize each of these parts as 1/b. <br> - represent fractions $a / b$ by a length of $1 / b$ from zero. |

Scope and Sequence
Grade 3

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 4 | 1. Comparing fractions on a number line. | 3.NF.A. 1 <br> 3.NF.A.2.b <br> 3.NF.A.3.a <br> 3.NF.A.3.c | Students: <br> - work with puzzles involving representing fractions as a bar model, moving the bar model to a number line, and understanding a fraction as a number on a number line. <br> - represent a fraction $1 / b$ on a number line by defining the interval from 0 to 1 and partitioning it into $b$ equal parts. Recognize each of these parts as $1 / b$. <br> - represent fractions $\mathrm{a} / \mathrm{b}$ by a length of $1 / \mathrm{b}$ from zero. <br> - use benchmark fractions and unit fractions to place fractions on a number line diagram. <br> - understand that two fractions are equivalent when they occupy the same point on a number line. <br> - express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. <br> - use the number line diagram to compare fractions. |
| 5 | 1. Comparing Fractions and Equivalent Fractions. | 3.NF.A.3.a <br> 3.NF.A.3.b. <br> 3.NF.A.3.c <br> 3.NF.A.3.d | Students: <br> - work with fraction strips to prove that two or more fractions are equivalent. <br> - compare fractions by seeing the relative length of the fraction pieces and by relating the fraction pieces to earlier number line work. <br> - reflect on their learning during summer school. |

## Ongoing Math Focus

## Student Discourse

## Students discuss:

- the role of the numerator and denominator in locating the fraction on the number line.
- fractions equivalent to 1
- how they know how many unit fractions to select.
- counting by unit fractions and how that compares to counting whole numbers
- equivalence.
- counting by unit fractions. How many unit fractions are needed to make one whole?
- the size and number of partitions created as the cursor moves from left to right in the puzzle.
- the meaning of the denominator in the fractions. Relate this to moving left to right on the number line.
- the role of the numerator in the fractions
- the need for each section of the rectangles to cover the same amount of area as the other sections in the rectangle in order for them to be the same fractional part.
- that each bar or number line model may not look alike or be the same size but it can represent the same number
- the role of the denominator in partitioning the number line and counting by unit fractions to locate a number on the number line.
- the number of unit fractions that are equal to one.
- the role of the numerator and denominator in locating a fraction on the number line.


## Student Problem-Solving Strategies

## Students:

- count the unit fractions as the fraction pieces roll out on the number line.
- continue to discuss the role of the numerator and denominator in the fractions and how that helps them solve the puzzles
- identify each section as a unit fraction and discuss how many unit fractions are in the whole.
- count the different number of sections by unit fractions and name that amount as an a/b fraction.
- determine strategies for partitioning a number line into fractions and estimating the location of a number on the number line by counting by unit fractions.
- determine how to use benchmark numbers to locate a fraction on the number line.
- use directional (right or left) comparison of fractions on the number line.


## Representations

## Students:

- write an equation to show the location on the number line the total as addition of unit fractions.
- compare how the circles on the Jiji cycle and the number line represent the number.
- write expressions or equations to represent the puzzles.
- write numbers as fractions greater than one and mixed numbers.
- build bar models with different manipulatives. Compare the bar models and discuss what number each bar model represents and where that number would be located on the number line.
- create a number line and partition it to locate the given fractions.


## Standards

## Number and Operations-Fractions

## Develop an understanding of fractions as numbers.

3.NF.A. 1 Understand a fraction $1 / b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a / b$ as the quantity formed by parts of size $1 / \mathrm{b}$.
3.NF.A. 2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.
3.NF.A.2.a Represent a fraction $1 / \mathrm{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1 / b$ and that the endpoint of the part based at 0 locates the number $1 / b$ on the number line.
3.NF.A.2.b Represent a fraction $a / b$ on a number line diagram by marking off a length $1 / b$ from 0 . Recognize that the resulting interval has size $a / b$ and that its endpoint locates the number $a / b$ on the number line.
3.NF.A. 3 Explain the equivalence of fractions in special cases and compare fractions by reasoning about their size.
3.NF.A.3.a Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
3.NF.A.3.b. Recognize and generate simple equivalent fractions, e.g., $1 / 2=2 / 4,4 / 6=2 / 3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
3.NF.A.3.c Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3=3 / 1$; recognize that $6 / 1=6$; locate $4 / 4$ and 1 at the same point of a number line diagram.
3.NF.A.3.d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>,=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Scope and Sequence
Grade 4

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 1 | 1. Learn the routines and expectations for ST Math Summer Immersion <br> 2. Begin working with problems involving fractions <br> 3. Compare fractions; equivalent fractions; and benchmark fractions <br> 4. Solve problems involving comparing fractions, including ordering on a number line. | 4.NF.A. 2 | Students: <br> - will develop an understanding of the Problem-Solving Process and apply this thinking as a strategy for solving puzzles in ST Math. Students will be engaged in conversations and practice that will teach them how to utilize the strategy. <br> - will begin solving problems involving fractions, including all 3 operations (+; -; $x$ ) and understanding fractions as numbers. <br> - will develop an understanding of the design process and understand the purpose and expectations of the stations. <br> - create a model of a number using paper strips, connecting cubes, or Cuisenaire rods, and build a number line using the model. <br> - use benchmark fractions, equivalent fractions, and comparing fractions to place fractions on the number line <br> - justify their reasoning for where they placed the numbers. |
| 2 | 1. Comparing fractions and understanding equivalence. | 4.NF.A. 1 <br> 4.NF.A. 2 | Students: <br> - create fractions using paper strips. <br> - use the strips and the number line they created to compare fractions and find equivalent fractions. <br> - solve problems involving comparing and ordering fractions. |
| 3 | 1. Adding and subtracting fractions and mixed numbers. | 4.NF.B.3.a <br> 4.NF.B.3.b <br> 4.NF.B.3.c <br> 4.NF.B.3.d | Students: <br> - work with area models and number line models to add and subtract fractions and mixed numbers. <br> - use a number line to represent addition and subtraction of fractions as jumps to the right or left. <br> - relate this to addition and subtraction of whole numbers. <br> - solve rich problems involving addition and subtraction of fractions. |

## Scope and Sequence

Grade 4

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 4 | 1. Multiplying fractions by whole numbers. | 4.NF.B.4.a <br> 4.NF.B.4.b | Students: <br> - work with puzzles involving the multiplication of a fraction by a whole number using area models and number line models. <br> - extend their understanding of multiplication to multiply a whole number by a fraction. <br> - solve rich problems involving multiplying a whole number by a fraction |
| 5 | 1. Write and compare decimal fractions. <br> 2. Represent decimal fractions on a number line and a hundred grid. | 4.NF.C. 5 4.NF.C. 6 4.NF.C. 7 | Students: <br> - work with a number line and a hundred grid to represent decimal fractions. <br> - compare decimal and fraction forms of numbers. <br> - discuss the relationship of tenths and hundredths. |

## Ongoing Math Focus

| Student Discourse | Student Problem Solving Strategies | Representations |
| :---: | :---: | :---: |
| Students discuss: <br> - the role of the denominator and numerator in placing numbers on a number line. <br> - the size of the fractions (denominator) and the number of unit fractions (numerator) of that size to compare equivalent fractions. <br> - for puzzles with fifths and tenths, discuss when there will and will not be an equivalent. <br> - what happens to the numerator and denominator of the fractions when they are partitioned further. <br> - fractions equal to one, how they know fractions are equivalent on the number line, benchmark fractions, and counting by unit fractions. <br> - the role of the numerator and denominator. <br> - what each model and number represents in the problem. <br> - the relationship of addition and subtraction. <br> - how to determine the number of partitions (denominator) needed in the pies. <br> - how to determine how many pies to select (whole number factor times the numerator). <br> - how these puzzles compare to the puzzles in Crank Pies Fraction Multiplication. (two different games) <br> - the relationship of tenths and hundredths. | Students: <br> - determine how to select the denominator to partition the number line. <br> - discover as the number of partitions increases the size of the partitions decreases. <br> - compare original unit fractions to the new fraction after it is partitioned. <br> - include the fraction ( $a / a$ ) that they multiply the original fraction by in order to get the equivalent fraction after it is cut. (e.g., $13 \times$ $44=412$ ) <br> - determine how to order fractions from least to greatest. <br> - use counting by unit fractions and the similarity to adding whole numbers to add and subtract fractions. <br> - show the action of adding and subtracting fractions as jumps on a number line. <br> - use the relationship of addition and subtraction to solve problems. <br> - compare multiplying a whole number by a whole number to multiplying a whole number times a fraction. <br> - compare multiplying a whole number times a unit fraction to multiplying a whole number times an $\mathrm{a} / \mathrm{b}$ fraction. | Students: <br> - create fractions by cutting paper strips. <br> - write equivalent fractions and mixed numbers. <br> - write equations to show the equivalence of fractions. <br> - Write equations and inequalities to compare fractions. <br> - create a number line and compare and order fractions on the number line. <br> - write fractions greater than 1, as a fraction, and as mixed numbers. <br> - use a bar model and a number line to add and subtract fractions. <br> - write equations to represent the problems. <br> - compare the action of multiplying a whole number times a fraction on a number line to the action of adding fractions on a number line. <br> - write addition and multiplication equations to represent the puzzles. <br> - represent the solutions on a hundred grid. <br> - use the models to show the relationship of tenths and hundredths. <br> - write decimal and fraction forms of the numbers. <br> - write equations to represent the numbers (e.g., $0.53=0.5+0.03$ ). |

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- compare fraction and decimal forms for writing the numbers. Change one form to the same form as the other addend to add.


## Standards

## Number and Operations-Fractions

## Extend understanding of fraction equivalence and ordering.

4.NF.A. 1 Explain why a fraction $a / b$ is equivalent to $a$ fraction $(n \times a) /(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.A. 2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

## Build fractions from unit fractions.

4.NF.B. 3 Understand $a$ fraction $a / b$ with $a>1$ as a sum of fractions $1 / b$.
4.NF.B.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
4.NF.B.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3 / 8=1 / 8+1 / 8+1 / 8 ; 3 / 8=1 / 8+2 / 8 ; 21 / 8=1+1+1 / 8=8 / 8+8 / 8+1 / 8$.
4.NF.B.3.c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
4.NF.B.3.d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.B. 4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
4.NF.B.4.a Understand a fraction $a / b$ as a multiple of $1 / b$. For example, use a visual fraction model to represent $5 / 4$ as the product $5 \times(1 / 4)$, recording the conclusion by the equation $5 / 4=5 \times(1 / 4)$.
4.NF.B.4.b Understand a multiple of $a / b$ as a multiple of $1 / b$ and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times(2 / 5)$ as $6 \times(1 / 5)$, recognizing this product as $6 / 5$. (In general, $n \times(a / b)=(n \times a) / b$.)
4.NF.B.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3 / 8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Scope and Sequence
Grade 4

## Standards

## Understand decimal notation for fractions and compare decimal fractions.

4.NF.C. 5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 For example, express $3 / 10$ as $30 / 100$, and add $3 / 10+4 / 100=34 / 100$.
4.NF.C. 6 Use decimal notation for fractions with denominators 10 or 100 . For example, rewrite 0.62 as $62 / 100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
4.NF.C. 7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>_{,}=$, or $<$, and justify the conclusions, e.g., by using a visual model.

| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 1 | 1. Learn the routines and expectations for ST Summer Math Immersion <br> 2. Begin working with problems involving fractions. <br> 3. Understand the relationship of fractions ( $1 / 2,1 / 3,1 / 4,1 / 5,1 / 10$ ) to decimals. | 5.NF.A. 1 <br> 5.NF.A. 2 | Students: <br> - will develop an understanding of the Problem-Solving Process and apply this thinking as a strategy for solving puzzles in ST Math. Students will be engaged in conversations and practice that will teach them how to utilize the strategy. <br> - will begin solving problems involving fractions, including all 4 operations, and understanding fractions as numbers. <br> - will develop an understanding of the design process and understand the purpose and expectations of the stations. |
| 2 | 1. Adding and Subtracting Fractions with Unlike Denominators. | 5.NF.A. 1 <br> 5.NF.A. 2 | Students: <br> - use benchmark fractions, equivalent fractions, and comparing fractions to create a number line to compare and order fractions, place them on a number line, and justify their reasoning for where they placed the numbers. <br> - use this to assess the reasonableness of answers as they add and subtract fractions with unlike denominators. |
| 3 | 1. Solving problems involving multiplying a fraction or a whole number by a fraction. | 5.NF.B.4.a <br> 5.NF.B.4.b <br> 5.NF.B.5.a <br> 5.NF.B.5.b <br> 5.NF.B. 6 | Students: <br> - work with area models and number line models to multiply fractions by whole numbers and other fractions. <br> - work with puzzles involving multiplication of a fraction by a whole number, a whole number by a fraction, and a fraction by a fraction. <br> - relate multiplication of whole numbers to multiplication with fractions. |
| 4 | 1. Solving problems involving dividing fractions | 5.NF.B. 3 <br> 5.NF.B.7.a <br> 5.NF.B.7.b <br> 5.NF.B.7. | Students: <br> - work with puzzles involving multiplication and division of fractions and whole numbers. <br> - work a series of puzzles that help them understand the relationship between multiplication and division. |

Scope and Sequence
Grade 5

## STMath.

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| Module | Objectives | Standards | Overview |
| :---: | :---: | :---: | :---: |
| 5 | 1. Solving problems involving dividing fractions. | 5.NF.B. 3 <br> 5.NF.B.7.a <br> 5.NF.B.7.b <br> 5.NF.B.7.c | Students: <br> - work with puzzles involving multiplication and division of fractions and whole numbers. <br> - work on a series of puzzles that help them understand the relationship between multiplication and division. |

## Ongoing Math Focus

## Student Discourse

Students discuss:

- the relationship of fractions with denominators of $2,3,4,5$, and 10 to decimals.
- how to compare fraction and decimal forms of numbers.
- the numerator determines the number of jumps (e.g., $3 / 4 \times 5$; 0 to 5 on the number line is partitioned into 4 equal sections; jiji jumps to the end of 3 of those sections).
- the role of the numerator and denominator in the visual representation and the multiplication expression.
- what happens to the size of the product when multiplying a fraction times a whole number and a fraction times a fraction (fractions less than and greater than one).
- what each number in the equation represents in the puzzle.
- the relationship between multiplication and division and the role of the numerator and denominator in determining the solution.


## Student Problem-Solving Strategies

Students:

- determine the number of unit fractions needed to equal the given decimal sum.
- recognize that there are 10 hundredths for every tenth ( $0.01 \times 10=0.1$ ).
- develop strategies for locating fractions on a number line by visually partitioning each whole into a number of sections based on the denominator and counting by unit fractions based on the numerator.
- develop an understanding of multiplying the numerator and denominator by the same number to find equivalent fractions.
- partition each whole on the number line into the appropriate number of partitions needed to locate the sum.
- understand that as the size of the partitions decreases the number of partitions increases to make one whole.
- determine how to partition the number line and determine the size and number of jumps to make when multiplying a fraction and whole number.
- multiply a fraction times a fraction using an area model.
- determine how many pies one monster would eat given a number of pies for 2 or more monsters. Then determine how many pies are needed to feed the given number of monsters.
- partition the pies on the game mat, and distribute the pies to the monsters.


## Representations

Students:

- record the sum on a hundred grid to compare tenths to hundredths.
- locate fraction form (1/10; 1/100) and decimal form ( $0.1 ; 0.01$ ) of numbers on a number line labeled 0 to 1 with tick marks for every tenth.
- locate fractions on number lines with and without partitions marked.
- write fractions greater than 1 as mixed numbers and fractions.
- understand that numbers that are located at the same point on a number line are equivalent.
- write mixed numbers as equivalent fractions.
- write equations to represent the visual problem.
- add and subtract mixed numbers and locate the sum on a number line.
- see the multiplication expression play out on the number line as taking the fraction the whole number of times (jumps) from left to right on the number line.
- solve multiplication problems with a unit fraction or an $\alpha / b$ fraction times a whole number using an area model.
- represent the problem on a number line and compare the two models.
- draw the area models before solving the puzzle online.

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- use the relationship of multiplication and division to
solve puzzles involving the division of whole numbers by fractions.
- represent the puzzles on their game mats and write equations to represent the puzzle.


## Standards

## Number and Operations-Fractions

## Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.A. 1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4=8 / 12+15 / 12=23 / 12$. (In general, $a / b$ $+c / d=(a d+b c) / b d$.
5.NF.A. 2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2 / 5+1 / 2=3 / 7$, by observing that $3 / 7<1 / 2$.

## Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B. 3 Interpret a fraction as a division of the numerator by the denominator $(a / b=a \div b)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3 / 4$ as the result of dividing 3 by 4 , noting that $3 / 4$ multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size $3 / 4$. If 9 people want to share a 50 -pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
5.NF.B.4.a Interpret the product $(a / b) \times q$ as parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2 / 3) \times 4=8 / 3$, and create a story context for this equation. Do the same with $(2 / 3) \times(4 / 5)=8 / 15$. (In general, $(a / b) \times(c / d)=a c / b d$.)
5.NF.B.4.b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.B. 5 Interpret multiplication as scaling (resizing), by:

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5.NF.B.5.a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

## Standards

5.NF.B.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number, and relating the principle of fraction equivalence $a / b=(n \times a) /(n \times b)$ to the effect of multiplying $a / b$ by 1 .
5.NF.B. 6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. 5.NF.B.7.a Interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for $(1 / 3) \div$ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( $1 / 3$ ) $\div 4=1 / 12$ because $(1 / 12) \times 4=1 / 3$.
5.NF.B.7.b Interpret division of a whole number by a unit fraction and compute such quotients. For example, create a story context for $4 \div(1 / 5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div(1 / 5)=20$ because $20 \times$ $(1 / 5)=4$.
5.NF.B.7.c Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1 / 2 \mathrm{lb}$ of chocolate equally? How many $1 / 3$ cup servings are in 2 cups of raisins?

